

FINITE DEFLECTIONS OF A SIMPLY SUPPORTED RIGID-PLASTIC ANNULAR PLATE LOADED DYNAMICALLY

NORMAN JONES

Division of Engineering, Brown University, Providence, Rhode Island

Abstract—A theoretical analysis is presented for the dynamic behavior of a simply supported rigid, perfectly plastic annular plate subjected to a rectangular pressure pulse. It is shown that this theory, which considers the simultaneous influence of membrane forces and bending moments, predicts final deformations which are considerably smaller than those given by the corresponding bending theory even when maximum deflections only of the order of the plate thickness are permitted. It is believed that this theoretical analysis could be developed further in order to describe the behavior of plates having other support conditions and different dynamic loading characteristics.

NOTATION

H	plate thickness
I	impulse per unit area of plate
I'	$I/(\mu H p_0)^{\frac{1}{2}}$
M_0	$\sigma_0 H^2/4$
M_r, M_θ	radial and circumferential bending moments per unit length
N_0	$\sigma_0 H$
N_r, N_θ	radial and circumferential membrane forces per unit length
Q	transverse shear force per unit length of plate
R	outside radius of plate
R_r, R_θ	principal radii of curvature
T	time at which plate reaches permanent position
a	inner radius of annular plate
$k(t), k_0, k_c$	uniform distributed pressure per unit area of undeformed plate
m_r, m_θ	dimensionless bending moments $M_r/M_0, M_\theta/M_0$
n_r, n_θ	dimensionless membrane forces $N_r/N_0, N_\theta/N_0$
n	defined by equation (19)
p	$-k \sin \phi$
p_0	$6M_0/R^2$
q	$-k \cos \phi$
r	radial coordinate of plate
t	time
u	displacement in direction r of undeformed plate
w	transverse deflection perpendicular to undeformed plate
α_r	$1 + \epsilon_r$
α_θ	$u + r$
δ	defined by equation (21)
$\epsilon_r, \epsilon_\theta$	radial and circumferential strains
ζ	k_0/k_c
θ	circumferential coordinate lying in plate
κ_r, κ_θ	radial and circumferential curvatures
λ	$\mu V_0^2 R^2/M_0 H$
μ	mass per unit area of plate
σ_0	yield stress in simple tension
τ	duration of pulse
ϕ	slope of the mid-plane of a plate measured in a plane which passes through $r = 0$ and is perpendicular to the plate surface

$$() \quad \frac{\partial}{\partial t}()$$

$$() \quad \frac{\partial}{\partial r}()$$

[] difference between the values of the considered quantity on either side of a travelling hinge

1. INTRODUCTION

THE behavior of rigid-plastic circular plates under the influence of static loads which produce infinitesimal deflections is fairly well established [1, 2, 3, etc.]. When finite deflections are permitted, however, it is observed that plates can support external loads considerably larger than those predicted by these theories. Onat and Haythornthwaite [4] indicated that this increased load carrying capacity is due mainly to the important role which membrane forces play in the finite deformation of plates.

It is clear from a survey of the pertinent literature that most attention has been directed towards the dynamic deformation of plates in which either membrane forces [5, 6, etc.] or bending moments [7, 8, 9, etc.] alone are believed to be important. Moreover, with the exception of some numerical work [10], the analysis of an annular plate by Florence [11], and some recent work [12], no investigations have been conducted into the interaction effects between membrane forces and bending moments, although such interaction influences considerably the static loading of plates [4] and the dynamic loading of beams [13]. Florence [14] applied uniform distributed impulses to some simply supported circular plates and observed that the appropriate rigid-plastic theory [8] overestimated considerably the recorded deflections particularly for large impulses. Recently it has been demonstrated [12] that a significant improvement in the theoretical predictions of plates loaded impulsively can be achieved if the influence of membrane forces and bending moments is retained in the theory.

Symonds [15] indicated that the permanent deformation of rigid-plastic beams subjected to central force pulses having rectangular and triangular shapes differed about $\pm 15\%$ from an equivalent half sine wave pulse with the same maximum value and impulse. Perzyna [16] developed further the theory of Hopkins and Prager [7], in which membrane forces are disregarded, and showed that for a given impulse the character of the pressure-time function had little influence on the final shape of a rigid-plastic circular plate. Hodge [17] and Sankaranarayanan [18], on the other hand, found that the blast characteristics had a profound effect upon the final deformation of cylindrical and spherical rigid-plastic shells.

In practice, the blast load which acts on a plate or structure often persists for a short period of time rather than behaving like a pure impulse as assumed in Ref. [12]. It is the purpose of this article, therefore, to study the behavior of a rigid, perfectly plastic annular plate when subjected to a rectangular pressure pulse such as the one shown in Fig. 1.

The limited interaction yield surface of Hodge [22], which retains all interaction between force and force, and moment and moment but disregards any interaction between forces and moments, will be used in the following analysis in order to simplify the various equations and permit an analytical solution. The results of this analysis will be compared with the corresponding values from the bending only theory so that they indicate the importance of membrane forces and with those when $\zeta \rightarrow \infty$ in order to examine and

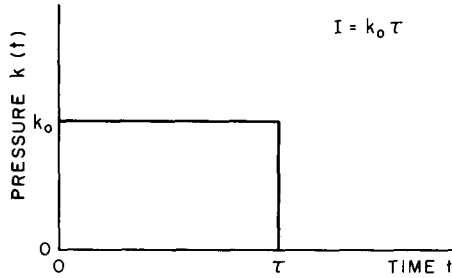


FIG. 1. Rectangular pressure pulse.

assess the difference between the permanent deflections corresponding to a pure impulse and an equivalent rectangular pressure pulse.

2. EQUILIBRIUM EQUATIONS

It may be shown that the equilibrium equations for the finite deflections of a circular plate subjected to axisymmetrical dynamic loads can be written in the form [12, 19]

$$(\alpha_\theta N_r)' - \alpha'_\theta N_\theta - \alpha_r \alpha_\theta Q / R_r + \alpha_r \alpha_\theta p + \mu \alpha_r \alpha_\theta \ddot{w} \sin \phi - \mu \alpha_\theta \alpha_r \ddot{u} \cos \phi = 0 \tag{1}$$

$$(\alpha_\theta Q)' + \alpha_r \alpha_\theta [N_r / R_r + N_\theta / R_\theta] + \alpha_r \alpha_\theta q + \mu \alpha_\theta \alpha_r \dot{w} \cos \phi + \mu \alpha_r \alpha_\theta \ddot{u} \sin \phi = 0 \tag{2}$$

$$(\alpha_\theta M_r)' - \alpha'_\theta M_\theta - \alpha_r \alpha_\theta Q = 0 \tag{3}$$

provided the rotary inertia effect is disregarded, and

$$\alpha_r = 1 + \epsilon_r$$

$$\alpha_\theta = r + u = r(1 + \epsilon_\theta)$$

$$1/R_r = \phi' / (1 + \epsilon_r)$$

$$1/R_\theta = \sin \phi / r.$$

The positive directions of the various quantities are indicated in Fig. 2.

If we limit our discussion to plates having small strains and deflections which are not too large, then we may let $\alpha_\theta = r$, $\alpha_r = 1$, $1/R_r = \phi'$, $1/R_\theta = \sin \phi / r$, and $\alpha'_\theta = \cos \phi$ which, using $\cos \phi = 1$, and $\sin \phi = -w'$, allow equations (1)–(3) to be recast as follows

$$rn'_r + n_r - n_\theta = -rkw' / N_0 + \mu r \ddot{w} w' / N_0 + \mu r \ddot{u} / N_0 \tag{4}$$

and

$$rm'_r + 2m'_r - m'_\theta - 4n_\theta w' / H = rk / M_0 - \mu r \ddot{w} / M_0 + \mu r \ddot{u} w' / M_0 \tag{5}$$

where,

$$n_r = N_r / N_0, \quad n_\theta = N_\theta / N_0$$

$$m_r = M_r / M_0, \quad m_\theta = M_\theta / M_0$$

and $r\phi'Q$, $rN_r\phi'$ and $\phi'w'$ have been disregarded.

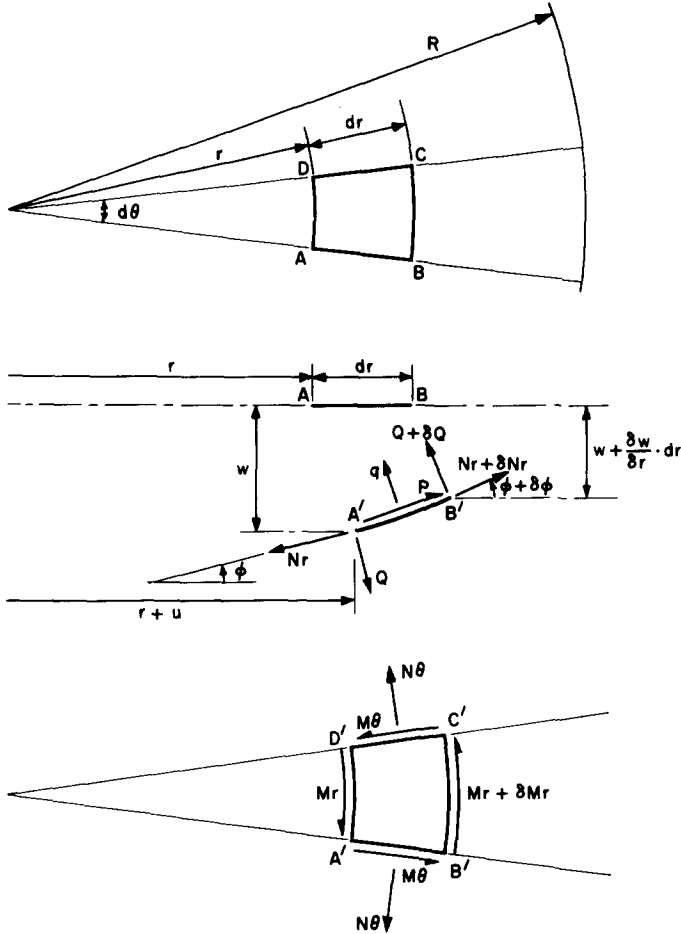


FIG. 2

3. STRAINS AND CURVATURES

It may be shown for small strains [19] that

$$\dot{\epsilon}_r = \dot{u}' + w' \dot{w}' \tag{6}$$

$$\dot{\epsilon}_\theta = \dot{u}/r \tag{7}$$

$$\dot{\kappa}_r = (1 + u') \dot{w}'' + \dot{u}' w'' - \dot{u}'' w' - u'' \dot{w}' \tag{8}$$

and

$$\dot{\kappa}_\theta = \dot{w}'/r \tag{9}$$

4. YIELD CONDITION

It has been found that disregarding elastic effects when analyzing cantilever beams loaded dynamically is a powerful simplification and a valid approximation, provided the

external energy is at least three times larger than the strain energy absorbed by the beam at the elastic limit [21]. Further, Frederick [5] and Boyd [6] investigated the deformation of membranes made from work-hardening material and found that a simplified perfectly plastic analysis provided a remarkably accurate model of the true behavior. Consequently the plate shown in Fig. 2 is assumed for the purposes of this analysis to be made from a rigid, perfectly plastic material.

The yield condition proposed by Hodge [22] and illustrated in Fig. 3 will be used in this article since it simplifies considerably a previous analysis, the results of which agree reasonably well with experimental values recorded on plates loaded impulsively [12]. This approximate yield surface is an "upper" bound to the Tresca yield condition for a uniform shell [23], while a similar one 0.618 times as large provides a "lower" bound.

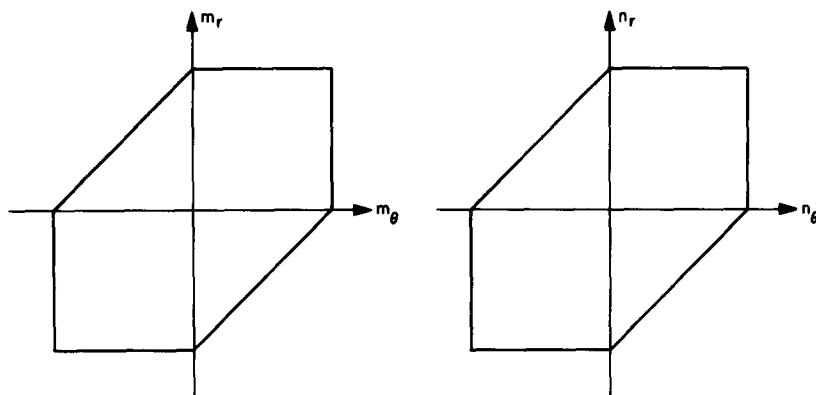


FIG. 3. Yield condition after Hodge [22].

5. FINITE DEFORMATION OF AN ANNULAR PLATE SUBJECTED TO A RECTANGULAR PRESSURE PULSE

In this section an examination is made of the behavior of a rigid, perfectly plastic annular plate subjected to the linearly distributed pressure pulse indicated in Fig. 4, the time variation of which is illustrated in Fig. 1. It is convenient to divide the analysis into two stages such that

$$k(t) = k_0 \frac{(R-r)}{(R-a)} \quad (10)$$

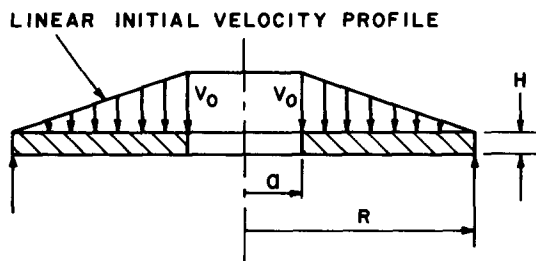


FIG. 4

during the first stage when $0 \leq t \leq \tau$, while

$$k(t) = 0$$

throughout the final stage, for which $\tau \leq t \leq T$.

5.1 First stage $0 \leq t \leq \tau$

The pressure distribution represented by equation (10) suggests that the transverse displacements might be of the form

$$w = W_1(t) \frac{(R-r)}{(R-a)} \tag{11}$$

where $W_1(t)$ is an unknown function of time and $a \leq r \leq R$.

Now, since it appears reasonable to assume that the radial strain ϵ_r is zero, then it may be shown, using equations (6) and (11), that

$$u = \frac{W_1^2(R-r)}{2(R-a)^2} \tag{12}$$

for an annular plate simply supported around its outer edge with $u = 0$ when $r = R$.

The strain and curvature rates given by equations (6)–(9), (11), and (12) satisfy the normality requirements associated with the yield surface illustrated in Fig. 3 when

$$n_\theta = 1, \quad 0 \leq n_r \leq 1 \tag{13}$$

and

$$m_\theta = -1, \quad -1 \leq m_r \leq 0 \tag{14}$$

Substituting equations (13) and (14) into (5) and disregarding $\ddot{u}w'$ when compared with \ddot{w} gives

$$\frac{M_0}{r^2} \frac{\partial}{\partial r} (r^2 m'_r) = k(t) - \mu \ddot{w} + \frac{N_0 w'}{r} \tag{15}$$

where $k(t)$ is an external pressure, $\mu \ddot{w}$ is an inertia term, and $N_0 w'/r$ arises from membrane forces which are introduced when finite deflections are permitted. It may be shown, when $a = 0$ and the $N_0 w'/r$ term is disregarded and either $k(t)$ or $\mu \ddot{w}$ or $k(t) - \mu \ddot{w}$ are retained, that equation (15) yields the same results as quoted in Refs. [1, 8, 7], respectively. If $k(t)$ and $N_0 w'/r$ are retained and $\ddot{w} = a = 0$, then equation (15) predicts results similar to those of Onat and Haythornthwaite [4] for deflections at $r = 0$ greater than $H/2$. The impulsive loading case in Ref. [12] was analyzed using equation (15), with $k(t) = 0$, while this article is concerned with dynamic loading for which all three terms must be included.

If equations (10) and (11) are substituted into (15), then

$$\frac{\partial}{\partial r} (r^2 m'_r) = -\frac{4W_1 r}{H(R-a)} + \frac{k_0(Rr^2 - r^3)}{M_0(R-a)} - \frac{\mu \ddot{W}_1(Rr^2 - r^3)}{M_0(R-a)} \tag{16}$$

which, when integrated twice with respect to r , gives

$$m_r = -\frac{2W_1}{H(R-a)} \left(r - 2a + \frac{a^2}{r} \right) + \frac{(k_0 - \mu \ddot{W}_1)}{M_0(R-a)} \left(\frac{Rr^2}{6} - \frac{Ra^2}{2} + \frac{Ra^3}{3r} - \frac{r^3}{12} + \frac{a^3}{3} - \frac{a^4}{4r} \right) + \frac{a}{r} - 1 \tag{17}$$

where the constants of integration have been evaluated from the conditions that $m_r = Q = 0$ at $r = a$.

Moreover, if the plate is simply supported around its outer edge, then $m_r = 0$ at $r = R$, and

$$\ddot{W}_1 + n^2 W_1 = -\delta' \quad (18)$$

where

$$n^2 = \frac{4p_0}{\mu H(1-\alpha)(1+3\alpha)} \quad (19)$$

$$p_0 = \frac{6M_0}{R^2} \quad (20)$$

$$\delta = \frac{n^2 H}{2} \quad (21)$$

and

$$\delta' = \delta \left\{ 1 - \frac{k_0(1-\alpha)(1+3\alpha)}{2p_0} \right\} \quad (22)$$

The solution of equation (18) is

$$W_1 = \frac{\delta'}{n^2} \cos nt - \frac{\delta'}{n^2} \quad (23)$$

where the constants of integration have been evaluated from the initial conditions $W_1 = \dot{W}_1 = 0$ when $t = 0$.

At the end of the first stage equation (23) gives

$$W_1 = \frac{\delta'}{n^2} \cos n\tau - \frac{\delta'}{n^2} \quad (24)$$

and

$$\dot{W}_1 = -\frac{\delta'}{n} \sin n\tau \quad (25)$$

5.2 Second stage $\tau \leq t \leq T$

The plate is now unloaded so that $k(t) = 0$ for $a \leq r \leq R$ and $\tau \leq t \leq T$.

Let

$$w = W_2(t) \frac{(R-r)}{(R-a)} \quad (26)$$

where $W_2(t)$ is an unknown function of time but must match equations (24) and (25) when $t = \tau$.

If the radial strain is zero, then equations (12)–(14) are again valid provided W_2 is substituted for W_1 . Thus equation (5) can be rewritten

$$\frac{\partial}{\partial r}(r^2 m_r') = -\frac{4W_2 r}{H(R-a)} - \frac{\mu \ddot{W}_2 (Rr^2 - r^3)}{M_0(R-a)} \quad (27)$$

the solution of which is

$$m_r = \frac{-2W_2}{H(R-a)} \left(r - 2a + \frac{a^2}{r} \right) - \frac{\mu \ddot{W}_2}{M_0(R-a)} \left(\frac{Rr^2}{6} - \frac{Ra^2}{2} + \frac{Ra^3}{3r} - \frac{r^3}{12} + \frac{a^3}{3} - \frac{a^4}{4r} \right) + \frac{a}{r} - 1 \quad (28)$$

where the constants of integration have been determined from the requirements that $m_r = Q = 0$ at $r = a$.

For an annular plate simply supported around the outer edge we have

$$\ddot{W}_2 + n^2 W_2 = -\delta \quad (29)$$

where n^2 and δ are defined by equations (19) and (21).

The solution of equation (29) is

$$W_2 = \left\{ \frac{(\delta - \delta')}{n^2} \cos n\tau + \frac{\delta'}{n^2} \right\} \cos n\tau + \frac{(\delta - \delta')}{n^2} \sin n\tau \sin n\tau - \frac{\delta}{n^2} \quad (30)$$

when W_2 and \ddot{W}_2 are made continuous with the corresponding values at the end of the first stage.

The plate reaches its final position when $\ddot{W}_2 = 0$, or

$$T = \frac{1}{n} \tan^{-1} \left\{ \frac{(\delta - \delta') \sin n\tau}{(\delta - \delta') \cos n\tau + \delta'} \right\} \quad (31)$$

and the permanent shape of the plate is

$$\frac{w_m}{H} = \frac{1}{2(1-\alpha)} \left\{ \sqrt{\left[\left(1 - \frac{\delta'}{\delta} \right)^2 + \left(\frac{\delta'}{\delta} \right)^2 + \frac{2\delta'}{\delta} \left(1 - \frac{\delta'}{\delta} \right) \cos n\tau \right]} - 1 \right\} \left(1 - \frac{r}{R} \right) \quad (32)$$

It may be shown that equation (4) is always satisfied for thin plates with $n_r = 0$ at $r = a$.

In order to ensure that m'_r remains positive at the outer edge of the plate, it is necessary that

$$\frac{W(t)}{H} \leq \frac{1+\alpha}{4\alpha} \quad (33)$$

while, for small values of α , the requirement that $m_r \geq -1$ demands a more severe limitation on the deflections. If one is interested in deflections, the magnitude of which would violate the inequality (33), then it appears reasonable to consider the analysis outlined here as valid up to the time $t = t_1$ when $m'_r = 0$ at $r = R$ while for $t_1 \leq t \leq T$ the plate could be considered to behave as a membrane.

If the deflections of the plate are assumed infinitesimal, then $n_r = n_\theta = 0$ and an analysis similar to the one presented gives equations (18) and (29) without the $n^2 W_1$ and $n^2 W_2$ terms, respectively. The permanent shape of the plate for this case may be shown to be

$$w_m = \frac{-\delta' \tau^2}{2(1-\alpha)} \left(1 - \frac{\delta'}{\delta} \right) \left(1 - \frac{r}{R} \right) \quad (34)$$

Moreover, one can show using equation (16) that the static collapse load “ k_c ” of an annular plate is

$$k_c = \frac{2p_0}{(1-\alpha)(1+3\alpha)} \quad (35)$$

provided finite deflections are not permitted.

It is convenient, therefore, to define a load factor

$$\zeta = \frac{k_0}{k_c}$$

which, using equations (22) and (35), becomes

$$\zeta = 1 - \frac{\delta'}{\delta} \quad (36)$$

Recasting equations (32) and (34) gives

$$\frac{w_m}{H} = \frac{1}{2(1-\alpha)} \left\{ \sqrt{\left[\zeta^2 + (1-\zeta)^2 + 2\zeta(1-\zeta) \cos \left(\sqrt{[(1-\alpha)(1+3\alpha)] \frac{I'}{\zeta}} \right) \right]} - 1 \right\} \left(1 - \frac{r}{R} \right) \quad (37)$$

and

$$\frac{w_m}{H} = \frac{(\zeta-1)(1+3\alpha)I'^2}{4\zeta} \left(1 - \frac{r}{R} \right) \quad (38)$$

where

$$I' = \frac{I}{(\mu H p_0)^{\frac{1}{2}}} \quad (39)$$

and

$$I = k_0 \tau \quad (40)$$

Finally, equations (37) and (38) reduce to the impulsive loading cases when $\zeta \rightarrow \infty$, or

$$\frac{w_m}{H} = \frac{1}{2(1-\alpha)} \left\{ \sqrt{\left[1 + \frac{\lambda(1-\alpha)(1+3\alpha)}{6} \right]} - 1 \right\} \left(1 - \frac{r}{R} \right) \quad (41)$$

and

$$\frac{w_m}{H} = \frac{\lambda(1+3\alpha)}{24} \left(1 - \frac{r}{R} \right) \quad (42)$$

where

$$\lambda = \frac{\mu V_0^2 R^2}{M_0 H} \quad (43)$$

and,

$$k_0 \tau = \mu V_0$$

6. DISCUSSION

It is clear from the results plotted in Fig. 5, particularly when deflections of the order of the plate thickness or greater are permitted, that membrane forces influence considerably

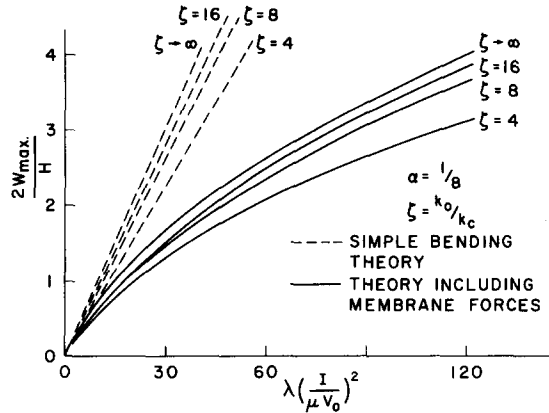


FIG. 5

the permanent deformation of a simply supported rigid, perfectly plastic annular plate loaded dynamically with a rectangular pressure pulse. Furthermore, deflections predicted for small values of ζ are significantly smaller than those for an equivalent impulse (i.e., $k_0\tau = \mu V_0$) although for $\zeta = 8$ the differences are about 10% and less than that for $\zeta > 8$.

7. CONCLUSIONS

A theoretical analysis which retains the influence of bending moments and membrane forces has been presented for a simply supported rigid-plastic annular plate loaded with a rectangular pressure pulse. It can be shown that this theoretical analysis predicts final deformations which are considerably smaller than those given by the bending only theory even for maximum deflections of the order of the plate thickness.

It is thought that the theoretical analysis presented here could be developed further in order to describe the behavior of plates having other support conditions and different characters of loading. However, it is believed that some estimate of the influence of strain-rate effects should be made since Wierzbicki [24] and Perrone [25] have shown that these are important in the bending only case.

Acknowledgements—The work reported herein was supported by the Advanced Research Project Agency, Department of Defense, under contract number SD-86 awarded to Brown University.

The author wishes to take this opportunity to express his appreciation to Miss E. Cerutti for computing the final results, and to the National Science Foundation (Grant Number GP-4825) for making funds available to cover the costs of machine time.

REFERENCES

- [1] H. G. HOPKINS and W. PRAGER, The load carrying capacities of circular plates. *J. Mech. Phys. Solids* **2**, 1-13 (1953).
- [2] D. C. DRUCKER and H. G. HOPKINS, Combined concentrated and distributed load on ideally-plastic circular plates. *Proc. 2nd U.S. Natn. Congr. appl. Mech.* 1954, pp. 517-520.
- [3] L. W. HU, Design of circular plates based on plastic limit load. *J. Engng Mech. Div. Am. Soc. civ. Engrs* **86**, 91-115 (1960).
- [4] E. T. ONAT and R. M. HAYTHORNTHWAITE, The load carrying capacity of circular plates at large deflection. *J. appl. Mech.* **23**, 49-55 (1956).

- [5] D. FREDERICK, A simplified analysis of circular membranes subjected to an impulsive loading producing large plastic deformations. *Proc. 4th Annual Conf. Solid Mech.*, University of Texas, Austin, Texas, Sept. 1959.
- [6] D. E. BOYD, Dynamic deformations of circular membranes. *J. Engng Mech. Div. Am. Soc. civ. Engrs* **92**, pp. 1–16 (1966).
- [7] H. G. HOPKINS and W. PRAGER, On the dynamics of plastic circular plates. *Z. angew. Math. Phys.* **5**, 317–330 (1954).
- [8] A. J. WANG, The permanent deflection of a plastic plate under blast loading. *J. appl. Mech.* **22**, 375–376 (1955).
- [9] A. L. FLORENCE, Clamped circular rigid-plastic plates under central blast loading. *Int. J. Solids Struct.* **2**, 319–335 (1966).
- [10] E. A. WITMER, H. A. BALMER, J. W. LEECH and T. H. H. PIAN, Large dynamic deformations of beams, circular rings, circular plates, and shells, *AIAA Launch and Space Vehicle Shell Struct. Conf.*, Palm Springs, California, April 1963.
- [11] A. L. FLORENCE, Annular plate under a transverse line impulse. *AIAA Jnl* **3**, 1726–1732 (1965).
- [12] N. JONES, Impulsive loading of a simply supported circular rigid-plastic plate. *J. appl. Mech.* **35**, 59–65 (1968).
- [13] P. S. SYMONDS and T. J. MENDEL, Impulsive loading of plastic beams with axial constraints. *J. Mech. Phys. Solids* **6**, 186–202 (1958).
- [14] A. L. FLORENCE, Circular plate under a uniformly distributed impulse. *Int. J. Solids Struct.* **2**, 37–47 (1966).
- [15] P. S. SYMONDS, Dynamic load characteristics in plastic bending of beams. *J. appl. Mech.* **20**, 475–481 (1953).
- [16] P. PERZYNA, Dynamic load carrying capacity of a circular plate. *Archwm Mech. stosow.* **10**, 635–647 (1958).
- [17] P. G. HODGE, The influence of blast characteristics on the final deformation of circular cylindrical shells. *J. appl. Mech.* **23**, 617–624 (1956).
- [18] R. SANKARANARAYANAN, On the dynamics of plastic spherical shells. *J. appl. Mech.* **30**, 87–90 (1963).
- [19] E. REISSNER, *Proc. Symp. Appl. Math.* **1**, 213–219 (1949).
- [20] J. GRIFFITH and H. VANZANT, Large deformation of circular membranes under static and dynamic loading, *1st Int. Congr. Exptl Mech.*, New York, Nov. 1961.
- [21] S. R. BODNER and P. S. SYMONDS, Experimental and theoretical investigation of the plastic deformation of cantilever beams subjected to impulsive loading. *J. appl. Mech.* **29**, 719–728 (1962).
- [22] P. G. HODGE, Yield conditions for rotationally symmetric shells under axisymmetric loading. *J. appl. Mech.* **27**, 323–331 (1960).
- [23] E. T. ONAT and W. PRAGER, Limit analysis of shells of revolution; Parts I and II. *Proc. R. Netherlands Acad. Sci.* **B57**, 534–541; 542–548 (1954).
- [24] T. WIERZICKI, Dynamics of rigid visco-plastic circular plates. *Archwm Mech. stosow.* **17**, 851–868 (1965).
- [25] N. PERRONE, Impulsively loaded strain-rate-sensitive plates. *J. appl. Mech.* **34**, 380–384 (1967).

(Received 19 June 1967; revised 27 December 1967)

Абстракт—приводится теоретический расчет динамического поведения свободно опертой, твердой, идеально пластической, кольцевой пластинки, подверженной импульсивному давлению на прямоугольнике. Показано, что эта теория, учитывающая совместное влияние мембранных усилий и изгибающих моментов, принимает во внимание конечные деформации. Они являются соответственно меньшими по сравнению с деформациями полученными из соответствующей теории изгиба Хопкинса и Прагера, даже когда допускаются максимальные изгибы порядка толщины пластинки. Предполагается, что теоретическое исследование можно развить дальше с целью описания поведения пластинок с другими граничными условиями и разными характеристиками динамической нагрузки.